

Azimuthal anisotropy and the distribution of linearly polarized gluons in DIS dijet production at high energy

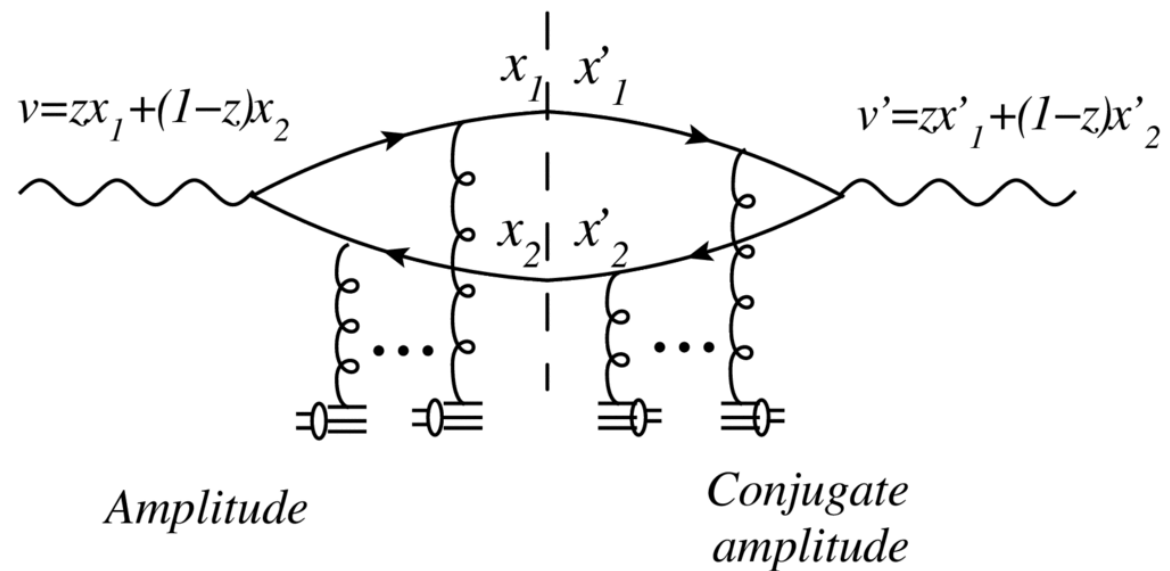
Adrian Dumitru
Baruch College, CUNY

CTEQ / POETIC 7
Temple University
Nov. 14 – 18, 2016

based on: A.D., T. Lappi, V. Skokov, 1508.04438 / PRL 115 (2015)
A.D., V. Skokov, 1605.02739 / PRD 94 (2016)

Dijets in $\gamma^* A$:

(Dominguez, Marquet, Xiao, Yuan,
PRD 2011)



CM tr. momentum:

$$\vec{P} = \frac{1}{2} (\vec{k}_1 - \vec{k}_2) \quad \text{or} \quad \tilde{P} = (1 - z)\vec{k}_1 - z\vec{k}_2$$

and momentum imbalance: $\vec{q} = \vec{k}_1 + \vec{k}_2$

- Both dijets in the hemisphere of γ^* ($y \geq 0$)

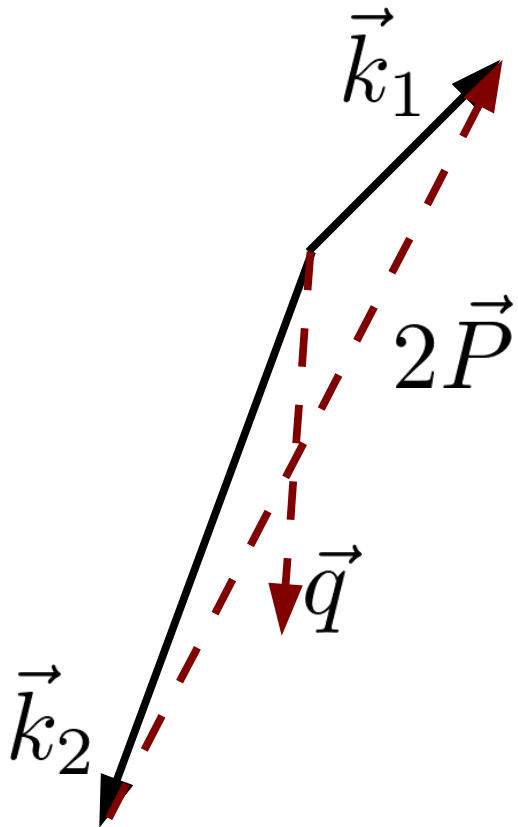
“correlation limit” $P \gg q$ involves only 2-point functions / TMDs (leading power)

Azimuthal anisotropy

(Dominguez, Qiu, Xiao, Yuan, PRD 2012
Boer, Mulders, Pisano, PRD 80 (2009) – 2016
Boer, Brodsky, Mulders, Pisano, PRL 106 (2011))

$$d\sigma^{\gamma_L^* A \rightarrow q \bar{q} X} = e_q^2 \alpha \alpha_s z^2 (1-z)^2 \frac{8\epsilon_f^2 \tilde{P}^2}{(\tilde{P}^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x, q) + \cos(2\phi) xh_{\perp}^{(1)}(x, q) \right)$$

ϕ = angle between \vec{P} and \vec{q}



→ rotate net transverse momentum vector q around and measure amplitude of $\cos(2\phi)$ modulation

$$v_2(q, x) = \langle \cos 2\phi \rangle = \frac{1}{2} \frac{h_{\perp}^{(1)}(x, q)}{G^{(1)}(x, q)}$$

The distribution of linearly polarized gluons

(in terms of L.C. gauge field correlator)

(Metz, Zhou: PRD 2011;
Dominguez, Qiu, Xiao,
Yuan,
PRD 2012)

$$xG_{\perp}^{(1)}(x, k) = -\frac{2}{\alpha_s L^2} \delta^{ij} \left\langle \text{Tr} \left[A_i(\vec{k}) A_j(-\vec{k}) \right] \right\rangle$$

$$xh_{\perp}^{(1)}(x, k) = \frac{2}{\alpha_s L^2} \left(\delta^{ij} - 2 \frac{k^i k^j}{k^2} \right) \left\langle \text{Tr} \left[A_i(\vec{k}) A_j(-\vec{k}) \right] \right\rangle$$

$$A_i(\vec{k}) = \int \frac{d^2 y}{(2\pi)^2} e^{-i\vec{k} \cdot \vec{y}} U^\dagger(\vec{y}) \partial_i U(\vec{y})$$

$$U(\vec{y}) = \mathcal{P} e^{-ig \int dz^+ A^-(z^+, \vec{y})}$$

$$\partial_i U(\vec{y}) = ig \int_{-\infty}^{\infty} dz^+ U(-\infty, z^+; \vec{y}) \partial_i A^-(z^+, \vec{y}) U(z^+, \infty; \vec{y})$$

We have computed these functions at small x
by solving JIMWLK from MV model initial conditions

F^{i-}

(A.D., T. Lappi, V. Skokov: 1508.04438)

Resummation of boost-invariant quantum fluctuations (JIMWLK):

classical ensemble at $Y = \log x_0/x = 0$:

$$P[\rho] \sim e^{-S_{\text{cl}}[\rho]}, \quad S_{\text{MV}} = \int d^2 x_{\perp} dx^+ \frac{1}{2\mu^2} \rho^a \rho^a, \\ V(x_{\perp}) = \mathcal{P} \exp ig^2 \int dx^+ \frac{1}{\nabla_{\perp}^2} \rho(x_{\perp})$$

JIMWLK quantum evolution: functional RG equation

$$\frac{\partial}{\partial Y} W[V] = -H \left[V, \frac{\delta}{\delta A^-} \right] W[V]$$

↑
distribution in space of Wilson lines

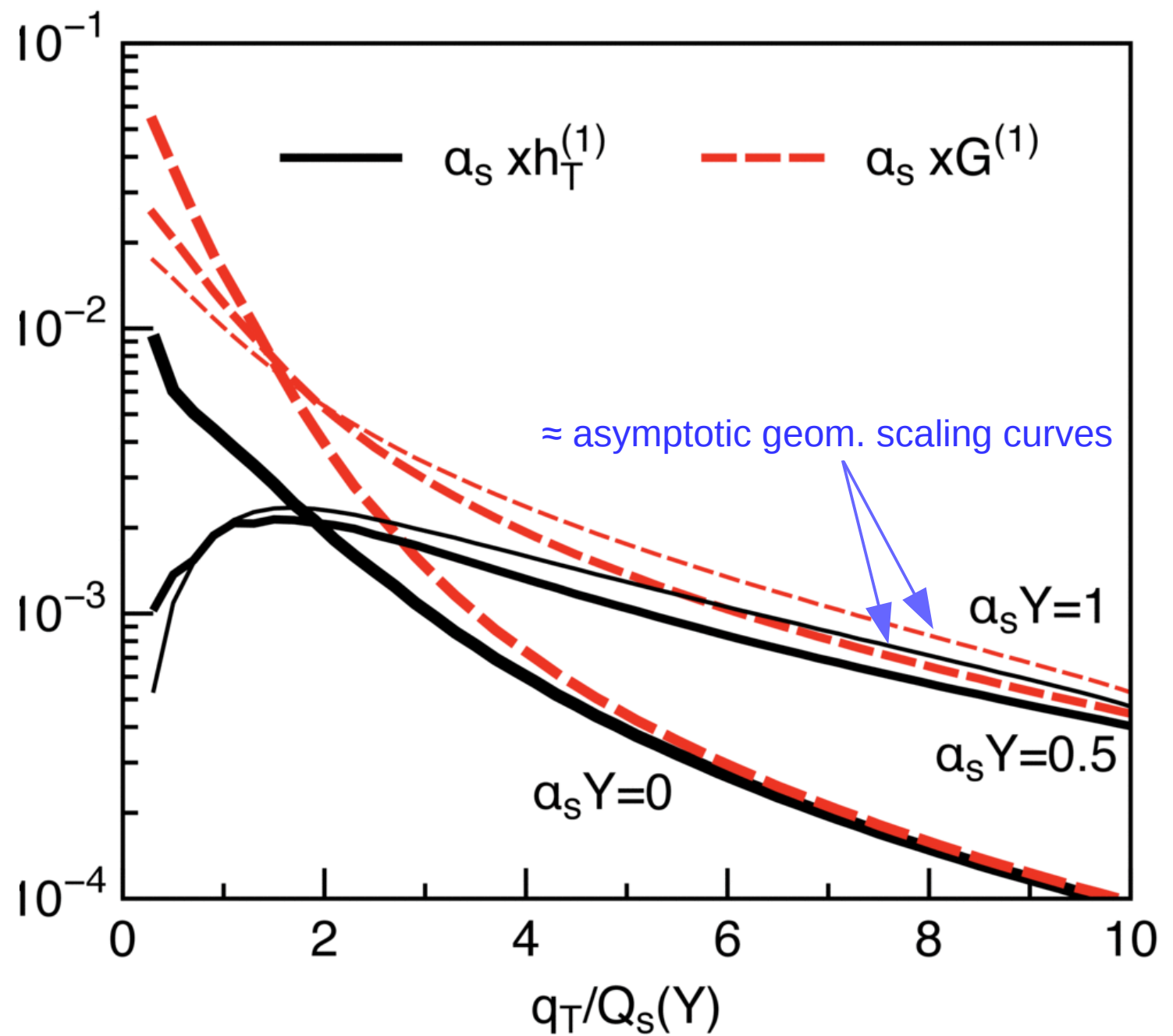
quantum evolution to $Y>0$: Langevin / random walk in space of Wilson lines

$$\partial_Y V(x_\perp) = V(x_\perp) \, it^a \left\{ \int d^2 y_\perp \, \varepsilon_k^{ab}(x_\perp, y_\perp) \, \xi_k^b(y_\perp) + \sigma^a(x_\perp) \right\} .$$

$$\varepsilon_k^{ab} = \left(\frac{\alpha_s}{\pi} \right)^{1/2} \frac{(x_\perp - y_\perp)_k}{(x_\perp - y_\perp)^2} \left[1 - U^\dagger(x_\perp) U(y_\perp) \right]^{ab}$$

$$\langle \xi_i^a(x_\perp) \, \xi_j^b(y_\perp) \rangle = \delta^{ab} \delta_{ij} \delta^{(2)}(x_\perp - y_\perp)$$

$$\sigma^a(x_\perp) = -i \frac{\alpha_s}{2\pi^2} \int d^2 z_\perp \frac{1}{(x_\perp - z_\perp)^2} \text{tr} \left(T^a U^\dagger(x_\perp) U(z_\perp) \right)$$

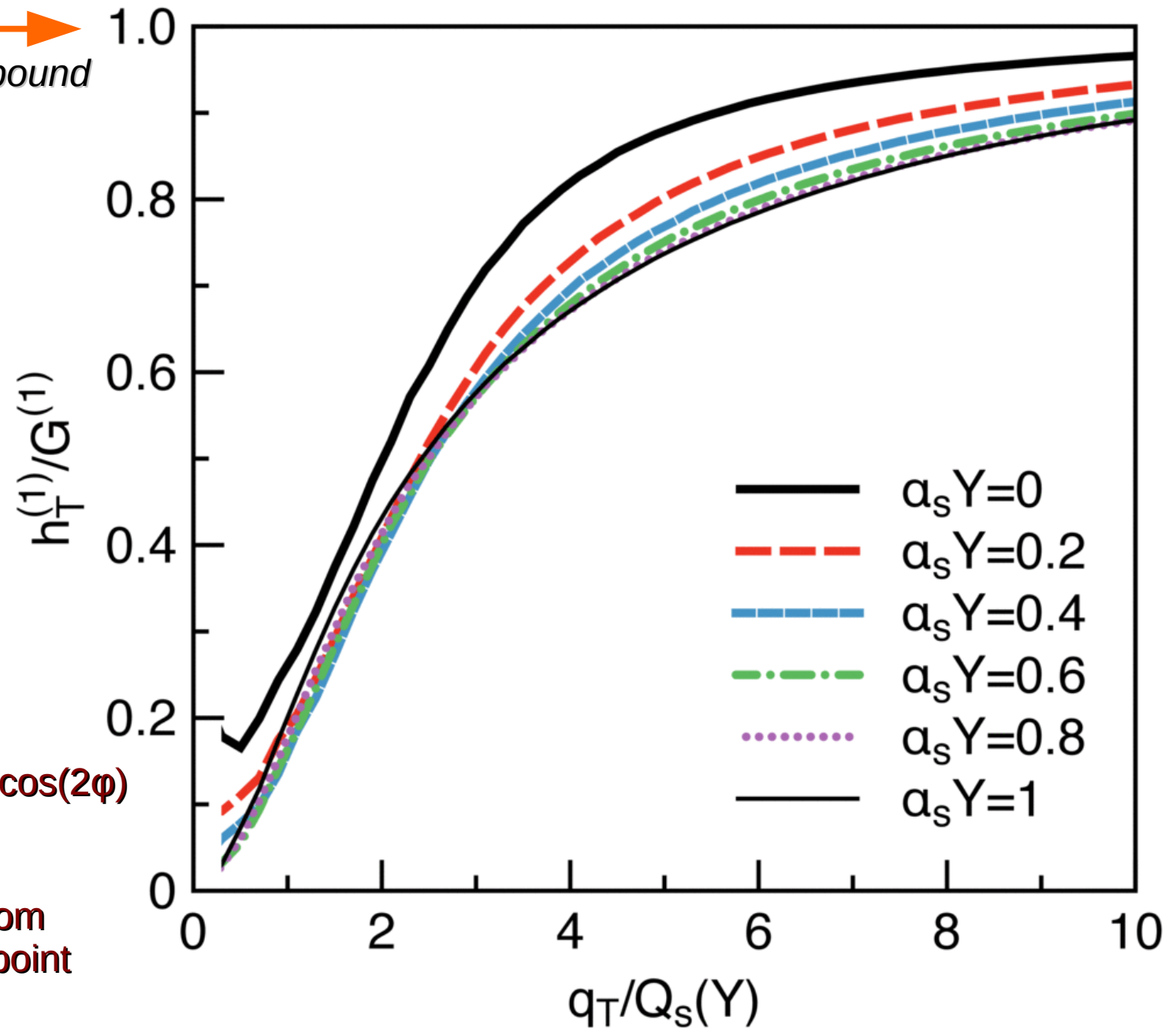


- $h_{\perp}^{(1)} / G^{(1)} \rightarrow 0$ at low q
- but $h_{\perp}^{(1)} / G^{(1)} \rightarrow 1$ at high transv. momentum:
saturates bound !

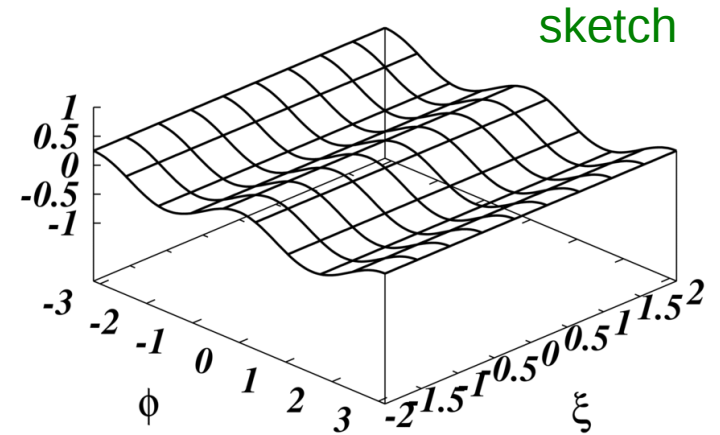
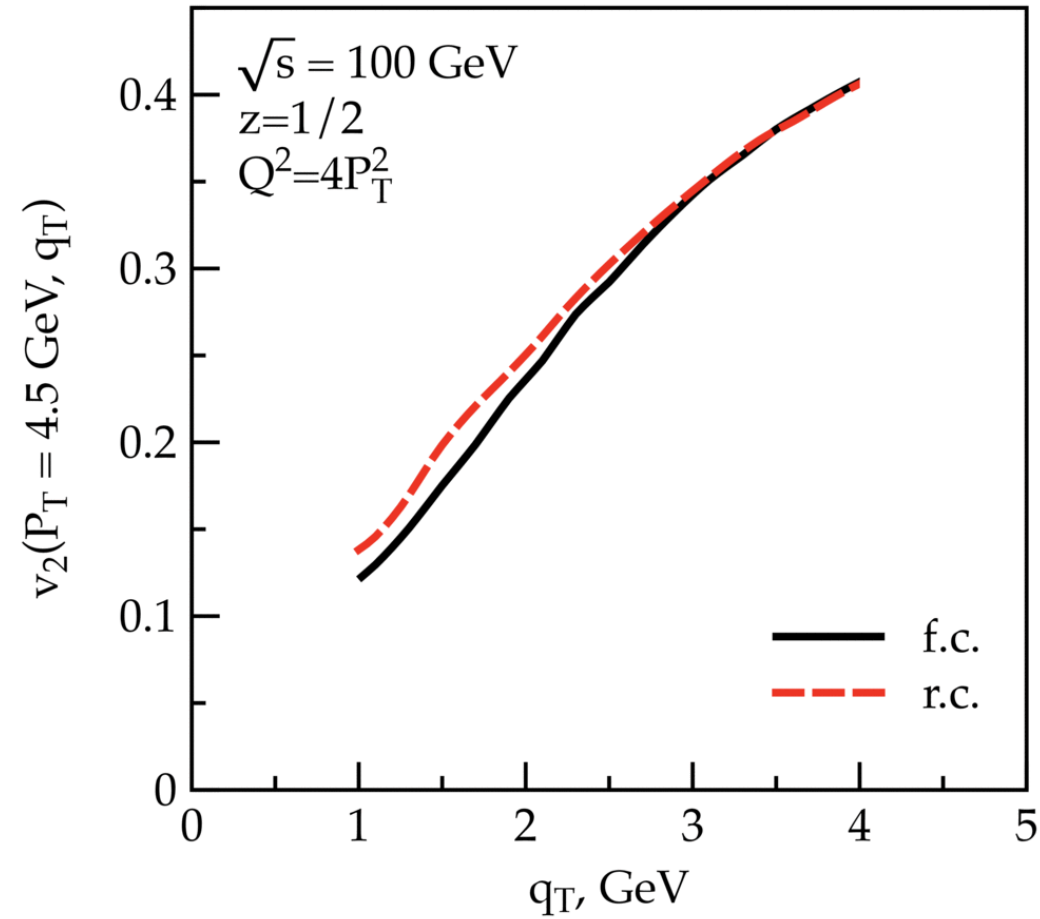


 saturation of positivity bound

- MV model gives large $\sim \cos(2\phi)$ anisotropy at $q_T > Q_s$
- rapid initial flow away from MV model \rightarrow RG fixed point
- followed by rather slow small- x evolution

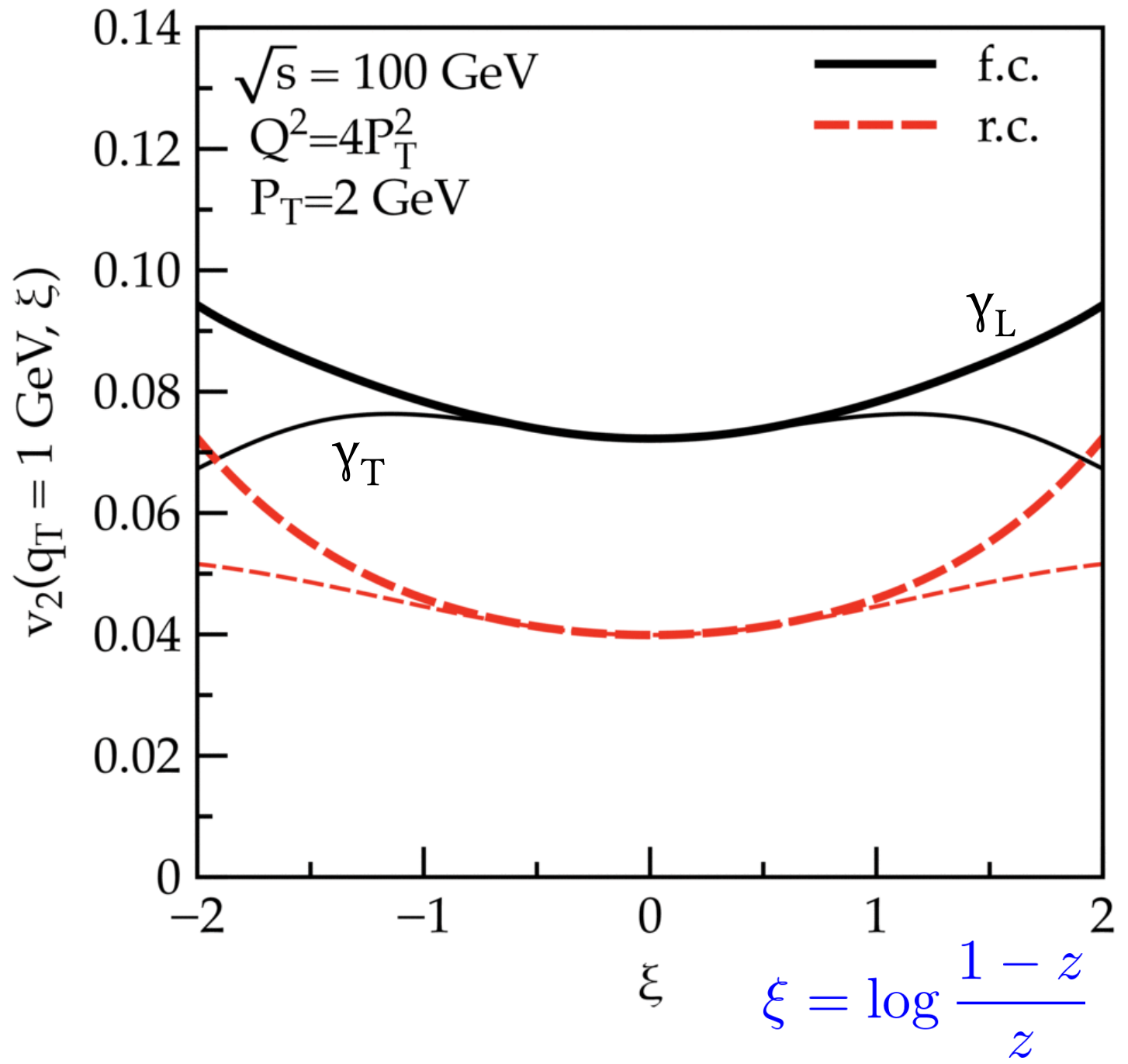


Large $\cos(2\phi)$ amplitudes...



$$\xi = \log \frac{1-z}{z}$$

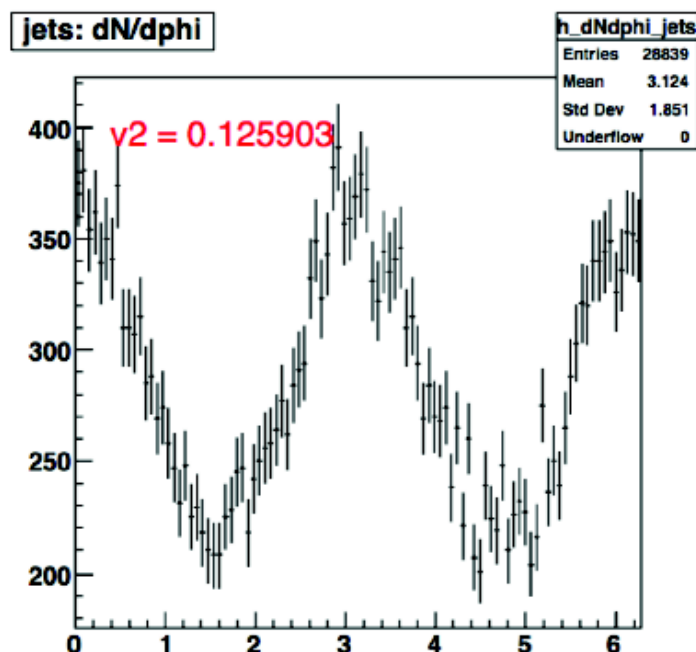
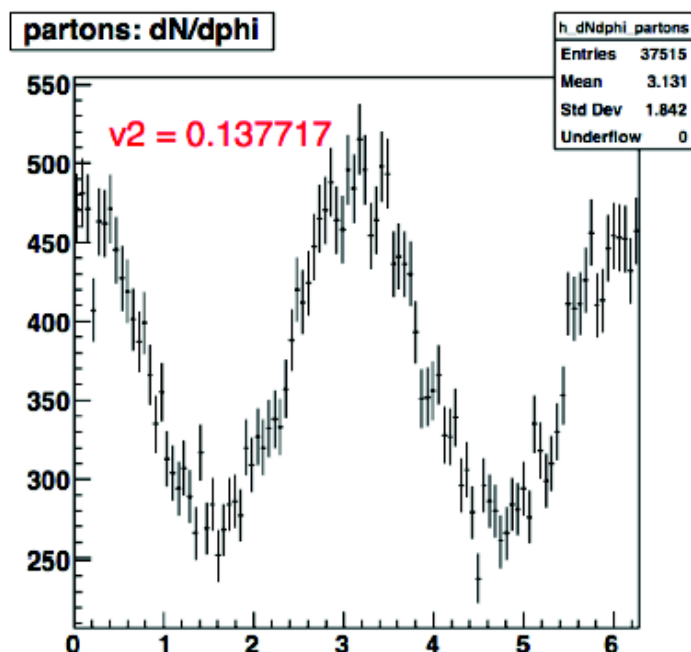
Amplitude of $\cos(2\Phi)$ is long range in rapidity



MONTÉ CARLO EVENT GENERATOR

- DIS event with random Q^2 , W^2 , photon polarization, as well as P_\perp and q_\perp
- Input: \sqrt{s} and A
- Q_s and target area are adjusted according to A
- Output: Parton 4-momentum etc
- Pythia afterburner \rightarrow particles
- Jet reconstruction

- $1 < q_t < 1.5$
- $2 < P_t < 2.5$
- $\text{pol}=1$ (L)



A. Dumitru, V. S. and T. Ullrich work in progress

$\cos 4\phi$ asymmetry beyond
leading power TMD

ok, back to the general expression for $\gamma^* A \rightarrow q\bar{q}X$

$$\frac{d\sigma^{\gamma^* A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} =$$

$$N_c \alpha_{em} e_q^2 \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2x_1}{(2\pi)^2} \frac{d^2x_2}{(2\pi)^2} \frac{d^2x'_1}{(2\pi)^2} \frac{d^2x'_2}{(2\pi)^2} e^{-i\vec{k}_1(\vec{x}_1 - \vec{x}'_1) - i\vec{k}_2(\vec{x}_2 - \vec{x}'_2)}$$

$$\sum_{\gamma\alpha\beta} \psi_{\alpha\beta}^{\text{T,L}}(\vec{x}_1 - \vec{x}_2) \psi_{\alpha\beta}^{\text{T,L}*}(\vec{x}'_1 - \vec{x}'_2)$$

$$\left[1 + \frac{1}{N_c} \left(\langle \text{Tr } U(\vec{x}_1) U^\dagger(\vec{x}_2) U(\vec{x}'_2) U^\dagger(\vec{x}'_1) \rangle \right. \right. \text{Quadrupole}$$

$$\left. \left. - \langle \text{Tr } U(\vec{x}_1) U^\dagger(\vec{x}_2) \rangle - \langle \text{Tr } U^\dagger(\vec{x}'_1) U(\vec{x}'_2) \rangle \right) \right]$$

write $e^{-i\vec{k}_1(\vec{x}_1 - \vec{y}_1) - i\vec{k}_2(\vec{x}_2 - \vec{y}_2)} = e^{-i\vec{P}(\vec{u} - \vec{u}') - i\vec{Q}(\vec{v} - \vec{v}')}$

$$\vec{u} = \vec{x}_1 - \vec{x}_2, \quad \vec{v} = (\vec{x}_1 + \vec{x}_2)/2$$

and expand in powers of u, u'

$$\begin{aligned}
\mathcal{Q}(\vec{x}_1, \vec{x}_2; \vec{x}'_2, \vec{x}'_1) &= 1 + \frac{\langle \text{Tr } U(\vec{x}_1) U^\dagger(\vec{x}'_1) U(\vec{x}'_2) U^\dagger(\vec{x}_2) \rangle - \langle \text{Tr } U(\vec{x}_1) U^\dagger(\vec{x}_2) \rangle - \langle \text{Tr } U(\vec{x}'_1) U^\dagger(\vec{x}'_2) \rangle}{N_c} \\
&= u_i u'_j \mathcal{G}^{i,j}(v, v') + u_i u'_j u'_k u'_l \mathcal{G}^{i,jkl}(v, v') + u_i u_j u_k u'_l \mathcal{G}^{ijk,l}(v, v') + u_i u_j u'_k u'_l \mathcal{G}^{ij,kl}(v, v') + \dots
\end{aligned}$$

$$\begin{aligned}
\mathcal{G}^{i,j}(v, v') &= -\frac{1}{N_c} \langle \text{Tr } V^\dagger(v) \partial_i V(v) V^\dagger(v') \partial_j V(v') \rangle \\
\mathcal{G}^{ij,mn}(v, v') &= \frac{1}{16N_c} \langle \text{Tr } [V^\dagger(v) \partial_i \partial_j V(v) + (\partial_i \partial_j V^\dagger(v)) V(v)] [(\partial_m \partial_n V^\dagger(v')) V(v') + V^\dagger(v') \partial_m \partial_n V(v')] \rangle , \\
\mathcal{G}^{ijm,n}(v, v') &= -\frac{1}{24N_c} \langle \text{Tr } [V^\dagger(v) \partial_i \partial_j \partial_m V(v) + 3(\partial_i \partial_j V^\dagger(v)) \partial_m V(v)] V^\dagger(v') \partial_n V(v') \rangle , \\
\mathcal{G}^{n,ijm}(v, v') &= -\frac{1}{24N_c} \langle \text{Tr } [V^\dagger(v) \partial_n V(v)] [V^\dagger(v') \partial_i \partial_j \partial_m V(v') + 3(\partial_i \partial_j V^\dagger(v')) \partial_m V(v')] \rangle
\end{aligned}$$

The dijet X-section involves the following combination:

$$\mathcal{G}^{ijmn}(x, q^2) = \mathcal{G}^{i,jmn}(x, q^2) + \mathcal{G}^{ijm,n}(x, q^2) - \frac{2}{3} \mathcal{G}^{ij,mn}(x, q^2)$$

introduce projectors \mathfrak{P}_1^{ijmn} , \mathfrak{P}_2^{ijmn} , \mathfrak{P}_3^{ijmn}
which project out $\cos(0\varphi)$, $\cos(2\varphi)$, $\cos(4\varphi)$

$$\Phi_2(x, q^2) = -\frac{2N_c}{\alpha_s} \mathfrak{P}_3^{ijmn} \mathcal{G}^{ijmn}(x, q^2)$$

explicit evaluation in Gaussian large- N_c model

$$Q^G = 1 + e^{-\frac{C_F}{2} [\Gamma(x_1 - x_2) + \Gamma(x'_2 - x'_1)]} - e^{-\frac{C_F}{2} [\Gamma(x_1 - x_2)]} - e^{-\frac{C_F}{2} [\Gamma(x'_2 - x'_1)]} \\ - \frac{\Gamma(x_1 - x'_1) - \Gamma(x_1 - x'_2) + \Gamma(x_2 - x'_2) - \Gamma(x_2 - x'_1)}{\Gamma(x_1 - x'_1) - \Gamma(x_1 - x_2) + \Gamma(x_2 - x'_2) - \Gamma(x'_2 - x'_1)} \\ \times \left(e^{-\frac{C_F}{2} [\Gamma(x_1 - x_2) + \Gamma(x'_2 - x'_1)]} - e^{-\frac{C_F}{2} [\Gamma(x_1 - x'_1) + \Gamma(x'_2 - x_2)]} \right)$$

Blaizot, Gelis, Venugopalan: hep-ph/0402257
Dominguez, Marquet, Xiao, Yuan: 1101.0715

perform same expansion in powers of u, u' :

$$xG^{(1)}(x, q^2) = \frac{4N_c}{\alpha_s} \frac{S_\perp}{(2\pi)^3} \int dr r J_0(qr) \left(1 - [S^{(2)}(r^2)]^2 \right) \left(\frac{\Gamma^{(1)}(r^2)}{\Gamma(r^2)} + r^2 \frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right) \\ xh^{(1)}(x, q^2) = \frac{4N_c}{\alpha_s} \frac{S_\perp}{(2\pi)^3} \int dr r^3 J_2(qr) \left(1 - [S^{(2)}(r^2)]^2 \right) \frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \\ \Phi_2(x, q^2) = -\frac{N_c}{\sqrt{2} 3\pi\alpha_s} \frac{S_\perp}{(2\pi)^2} \int dr J_4(rq) r^5 \\ \times \left[\frac{\Gamma^{(4)}(r^2)}{\Gamma(r^2)} \left(1 - [S^{(2)}(r^2)]^2 \right) - 5 \left(\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right)^2 \left[1 - [S^{(2)}(r^2)]^2 (1 + C_F \Gamma(r^2)) \right] \right]$$

small-x fixed point: $\Gamma(r^2) \sim (r^2 Q_s^2(x))^{\gamma_c}$

anomalous dimension $\gamma_c \sim 0.63$ near saturation boundary

$$\chi(\gamma_c)/\gamma_c = \chi'(\gamma_c)$$

Mueller & Triantafyllopoulos (2002)
S. Munier & R. Peschanski (2005)

$\gamma = 1 - O(\alpha_s)$ in DGLAP regime

more generally: when $S^{(2)}(r, x) = S^{(2)}(r Q_s(x))$

$xG^{(1)}(x, q)$, $xh^{(1)}(x, q)$, and $\Phi_2'(x, q) \equiv \Phi_2(x, q)/q^2$ exhibit “geometric scaling”,

i.e. functions only of $q/Q_s(x)$

DIJET CROSS SECTION

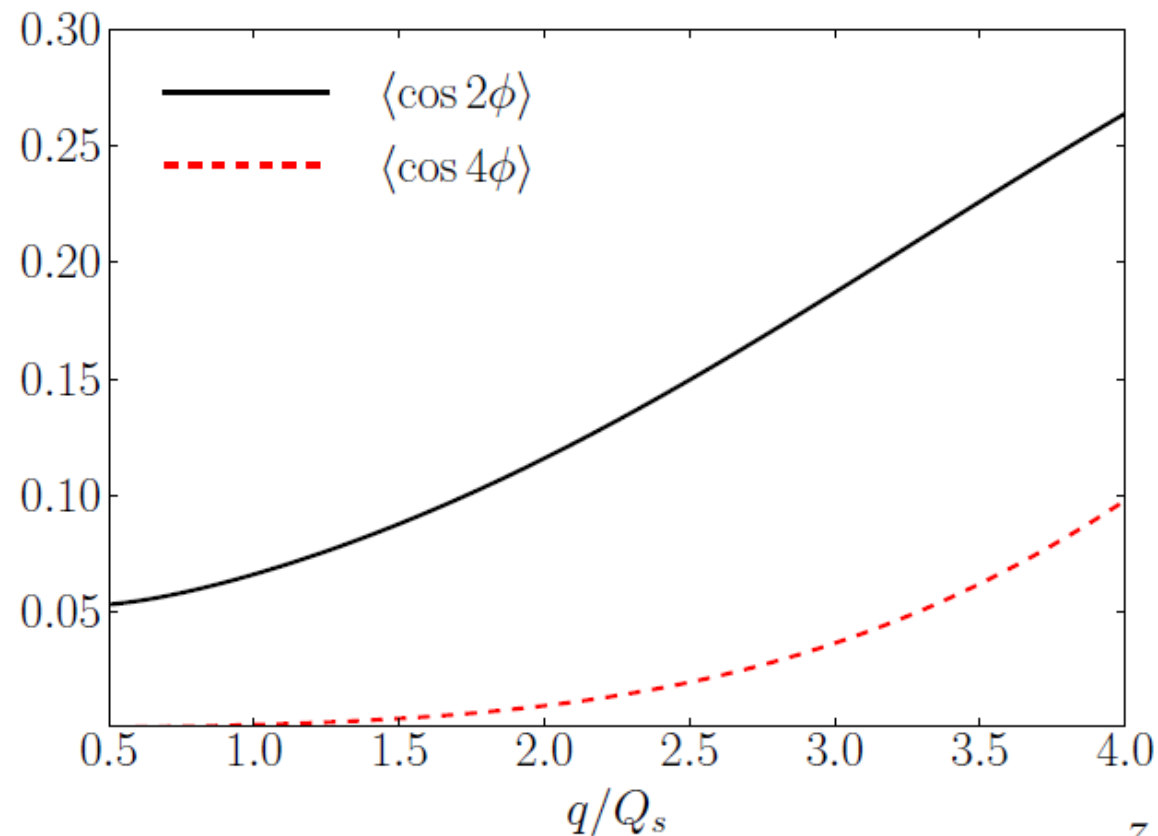
DiJet cross section to this order

$$\begin{aligned}
 & \frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} \\
 &= \alpha_s \alpha_{em} e_q^2 (z_1^2 + z_2^2) \left[\frac{P^4 + \epsilon_f^4}{(P^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x, q^2) - \frac{2\epsilon_f^2 P^2}{P^4 + \epsilon_f^4} xh^{(1)}(x, q^2) \cos 2\phi + O\left(\frac{1}{P^2}\right) \right) \right. \\
 & \quad \left. - \frac{48\epsilon_f^2 P^4}{\sqrt{2} (P^2 + \epsilon_f^2)^6} \Phi_2(x, q^2) \cos 4\phi \right] \\
 & \frac{d\sigma^{\gamma_L^* A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} \\
 &= 8\alpha_s \alpha_{em} e_q^2 z_1 z_2 \epsilon_f^2 \left[\frac{P^2}{(P^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x, q^2) + xh^{(1)}(x, q^2) \cos 2\phi + O\left(\frac{1}{P^2}\right) \right) \right. \\
 & \quad \left. + \frac{48P^4}{\sqrt{2} (P^2 + \epsilon_f^2)^6} \Phi_2(x, q^2) \cos 4\phi \right] .
 \end{aligned}$$

$$\begin{aligned}
 \text{DIS : } \epsilon_f^2 &= z_1 z_2 Q^2 \\
 Q^2 &\sim P^2
 \end{aligned}$$

A. Dumitru and V. S., *arXiv:1605.02739*

$\langle \cos 2\phi \rangle$ and $\langle \cos 4\phi \rangle$ in $\gamma_L^* + A \rightarrow q + \bar{q}$ dijet production from MV model:



$$z = 1/2, P = 4.5Q_s$$

A. Dumitru and V. S., arXiv:1605.02739

Summary:

- Dijet production in eA probes WW gluon TMD (in $P_T \gg q_T$ limit)
- WW distribution can be decomposed into **two** UGDs / TMDs
 - i) isotropic gluon probability $xG^{(1)}(x, q_T)$
 - ii) $\sim \cos(2\Phi)$ anisotropic distribution $xh^{(1)}(x, q_T)$ for orthogonal polarizations in amplitude vs. conjugate amplitude
- MV model gives large $\sim \cos(2\Phi)$ anisotropies at $q_T > Q_s$
- JIMWLK small- x evolution: $xG^{(1)}(x, q_T)$ and $xh^{(1)}(x, q_T)$ evolve similarly, (\sim geometric scaling regime), ratio drops slowly with Y
- this would result in “ridge”-like structure in terms of azimuthal angle of \vec{q}
- long-range in rapidity asymmetry $\xi = \log(1-z)/z$
- beyond leading power TMD: more involved operators w/ more derivatives; higher $\cos(2n\phi)$ components